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A detailed study of rough edge crack with worn asperities

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Abstract

A boundary element model which describes the reduced mode II and enhanced mode I stress intensity factors of rough edge crack problem with worn asperities is enforced in this paper. The dilatant boundary conditions (DBC) are assumed to be idealized uniform sawtooth crack surface and effective Coulomb sliding law. The boundary conditions of the closed worn edge crack are well described. The resulting COD, CSD and the stress distributions along the interface show the interference of a rough DBC edge crack surface. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Edge crack; Crack opening displacement (COD); Crack sliding displacement (CSD)

1. Introduction

Abrasive wear can be defined as the surface damage caused by two surfaces moving past each other while in contact as described in Fig. 1. The reduced mode II and the enhanced mode I stress intensity factors of the center crack problem (with elastic crack tip and plastic crack tip (based on Dugdale, 1960 and Becker and Gross, 1988) and the edge crack problem, (with worn asperities), were presented for an idealized sawtooth crack surface as in Young (1998) and also supported experimentally by the two papers of Tong et al (1995a, b). However, a further investigation about the COD, CSD and the stress distributions along the interface is still needed to have a clearer view about the interference of the rough edge crack surface. In the simple model, wear occurs once the macroscopic tangential resistance stress τ_c on the macro crack plane of the uniform sawtooth contact asperities reaches the yield stress in shear τ_y, i.e., $\tau_c = \tau_y = \mu\sigma_c$ and $\tau_y = \sigma_y/\sqrt{3}$, where σ_c and σ_y are the macroscopic normal resistance stress and the normal yield stress, respectively as described in Young (1998). The asperity is proposed to smear over, therefore, it causes a constant crack opening displacement CODW as shown in Fig. 1. The smear

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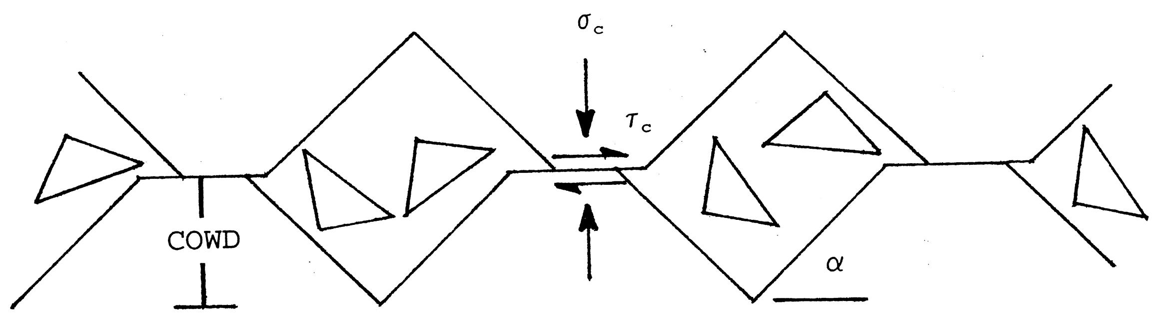

Fig. 1. Two dimensional geometry of idealized sawtooth wear with asperity angle α, crack opening displacement CODW, and coefficient of friction μ.

actually is an irreversible process consisting of either plastic deformation or fracture of asperities. The edge crack problem, i.e., a finite length crack is the half-plane, $x_1 > -a$, with uniform sawtooth surfaces, under mode II loading, as shown in Fig. 2, will be discussed here.

2. Closed worn crack

As discussed in Young (1998), the first step in the BEM solution is to divide the cracked homogeneous medium into two bodies (B_1 and B_2) along the plane of the crack referred to below as the interface, as shown in Fig. 2. The interaction between the two bodies is included through boundary

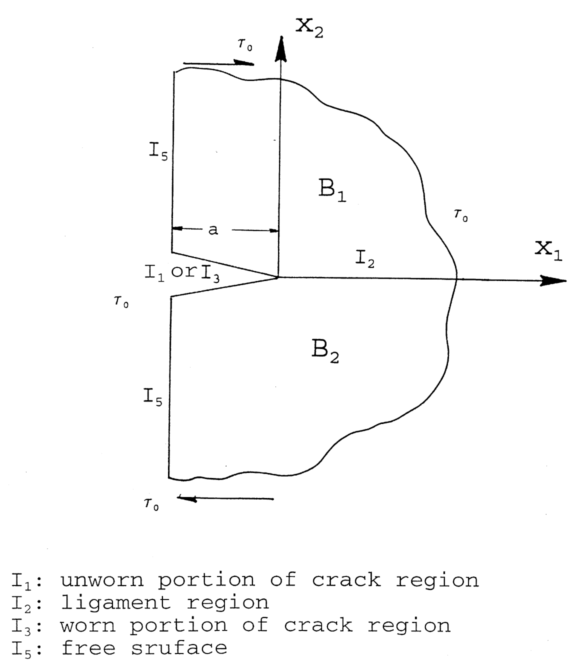

Fig. 2. Shear loading of idealized sawtooth edge crack with partial asperities worn in half infinite media.

conditions relating the displacements and stresses on either side of the interface. The truncated interface can be subdivided into three regions, i.e., an unworn DBC portion of crack region (I_1) as described in Young (1998), a ligament region (I_2), a worn portion of crack region (I_3). The truncated free surface of the half-plane is denoted by I_5 as shown in Fig. 2. Let ${}_\gamma t_i$ and ${}_\gamma u_i$ ($\gamma = 1$, 2 and $i = 1$, 2) denote the ith boundary traction and displacement components, respectively, on the boundary of B_γ. At points on the ligament region (I_2) the 2 displacement components and 2 stress components must be continuous. Therefore, the boundary conditions of this region are $({}_2t_1)_{I_2} = -({}_1t_1)_{I_2}$, $({}_2t_2)_{I_2} = -({}_1t_2)_{I_2}$, $({}_2u_1)_{I_2} = ({}_1u_1)_{I_2}$, and $({}_2u_2)_{I_2} = ({}_1u_2)_{I_2}$. This leaves $({}_1t_1)_{I_2}$, $({}_1t_2)_{I_2}$, $({}_1u_1)_{I_2}$, and $({}_1u_2)_{I_2}$, as the unknowns. At points on the unworn DBC portion of the crack region (I_1) the stresses are continuous (2 conditions) and are related by $\tau_c = \Gamma\sigma_c$ (1 condition), and $g = h\tan(\alpha)$ after g and h are expressed in terms of the displacements on opposite crack face, where, $\Gamma = (\mu + \tan\alpha)/(1 + \mu\tan\alpha)$ is the effective coefficient of friction, α is the asperity angle, g is the crack opening displacement, COD, and h is the crack sliding displacement, CSD as in Young (1998). Rewriting these two equations in terms of ${}_\gamma t_i$ and ${}_\gamma u_i$ and rearranging give the DBC, along with the usual continuity of stress $({}_2u_1)_{I_1} = ({}_1u_1)_{I_1}$ cot $\alpha({}_1u_2)_{I_1} - ({}_2u_2)_{I_1}$, $({}_2t_1)_{I_1} = -({}_1t_1)_{I_1}$, $({}_2t_2)_{I_1} = -({}_1t_2)_{I_1}$, and $({}_1t_2)_{I_2} = (1/\Gamma)({}_1t_1)_{I_1} + (\sigma_0 + (1/\Gamma)\tau_0)$, where, σ_0 is the applied normal stress which is zero in this case, and τ_0 is the applied shear stress. The unknowns are $({}_1t_1)_{I_1}$, $({}_1u_1)_{I_1}$, $({}_1u_2)_{I_1}$ and $({}_2u_2)_{I_1}$. As shown in Fig. 1 the asperities are assumed to begin to wear out when the macroscopic tangential resistance stress, τ_c, reaches the yield stress τ_y, i.e., $\tau_c = \tau_y = \mu\sigma_c$, $\tau_y = \sigma_y/\sqrt{3}$, and COWD is the value of COD at that point right before wear. In terms of ${}_\gamma t_i$ and ${}_\gamma u_i$ this gives the wear boundary conditions of the worn portion of crack (I_3), i.e., $({}_1t_2)_{I_3} = (1/\mu)({}_1t_1)_{I_3} + (\sigma_0 + (1/\mu)\tau_0)$, $({}_2t_2)_{I_1} = (1/\mu)({}_1t_1)_{I_3} - (\sigma_0 + (1/\mu)\tau_0)$, $({}_2t_1)_{I_3} = -({}_1t_1)_{I_3}$, and $({}_2u_2)_{I_3} = ({}_1u_2)_{I_3} - \text{CODW}$. The unknowns therefore, are $({}_1u_1)_{I_3}$, $({}_1u_2)_{I_3}$, $({}_1t_1)_{I_3}$, and $({}_2u_1)_{I_3}$. The boundary conditions of the crack region will shift from I_1 to I_3 once an asperity begins to wear. Thus, at each pair of points on the ligament region (I_2) we have four conditions involving eight quantities. Four of those are eliminated algebraically using the boundary conditions, thus, leaving four unknowns at each point. Two coupled boundary integral equations, written as a function of position on the boundary of a body, enforce all of the field equations of elasticity for that body. The two equations for each of the two artificially divided bodies are applied to each discretized point on the interface, thus giving four equations and four unknowns at each pair of interface points. At each of the boundary points of either of the artificially divided bodies consists of other than the common interface, i.e., the crack regions I_1 and I_3, there are four boundary quantities to be accounted for. The only condition we have used on free surface (I_5) has been prescribed stress, thus, leaving the two displacements as unknowns, with two equations provided by that body's two boundary integral equations, i.e., $({}_1t_1)_{I_5} = ({}_1t_2)_{I_5} = ({}_2t_1)_{I_5} = ({}_2t_2)_{I_5} = 0$. The unknowns are, of course, $({}_1u_1)_{I_5}$, $({}_1u_2)_{I_5}$, $({}_2u_1)_{I_5}$, and $({}_2u_2)_{I_5}$. The BEM consists of the discretization of the boundary surfaces and the numerical approximation of the boundary quantities in the set of equations obtained from the boundary integrals in Young (1994). We model the boundary, using straight-line elements, centered about nodes at which the integral equations are applied. The stress and displacement are assumed to be constant throughout each straight-line element. Therefore, the approximation allows their removal from the integral, resulting in integrals of the known 2D Green's function which have been evaluated in closed form in Young (1994). The final system of simultaneous linear algebraic equations for the unknown nodal displacements and stresses, can be obtained by using the Gaussian elimination method.

Figs. 3 and 4 show the stress distributions of the crack portion and the ligament portion, respectively, for aluminum with a remote uniform distribution of applied shear stress, $\tau_0 = 200$ MPa. The curves marked 'before' are for τ_0 just less than 200 MPa, which is the critical τ_0 for yield to occur, and the curves marked 'after' are for $\tau_0 = 200$ MPa after wear has occurred. It seems that the crack wears thoroughly when all asperities reach the shear yield stress τ_y simultaneously due to the uniformity of the applied load. Both normal and shear stress obviously decrease after wear. However, in the ligament

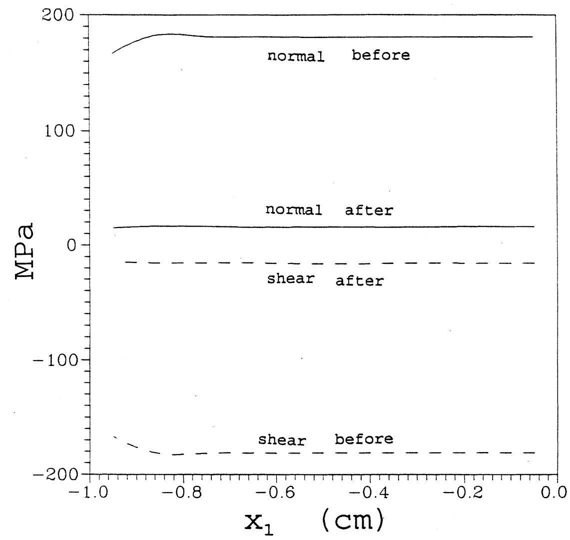

Fig. 3. Stress distributions in crack portion of edge crack problem in aluminum with worn asperities, 10 mm crack length, 200 MPa applied shear stress, 40° asperity angle, $\nu = 0.3$, G = 27000 MPa. Crack is between $x_1 = 0$ cm and −1 cm.

portion the normal stress is unchanged before and after wear, while the shear stress is obviously reduced.

Fig. 5 shows the COD and CSD before and after the wear occurs for aluminum with a remote uniform distribution of applied shear stress, $\tau_0 = 200$ MPa. The curves marked 'before' are for τ_0 just less than 200 MPa, which is the critical τ_0 for yield to occur, and the curves marked 'after' are for $\tau_0 = 200$ MPa after wear occurs. It seems that the crack wears totally when all asperities reach the shear

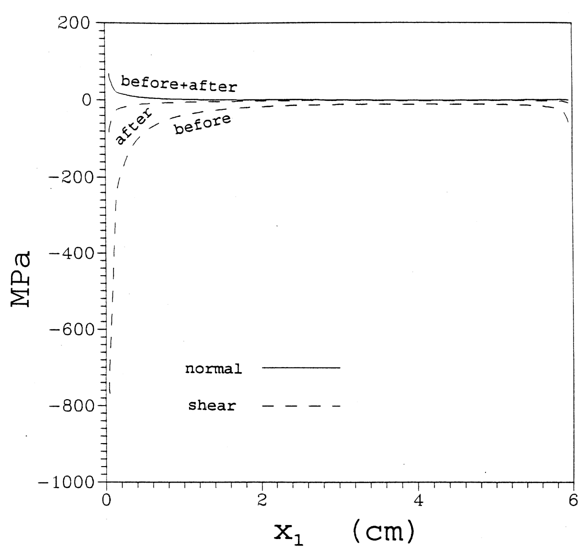

Fig. 4. Stress distributions in ligament portion of edge crack problem in aluminum with worn asperities, 10 mm crack length, 200 MPa applied shear stress, 40° asperity angle, $\nu = 0.3$, G = 27000 MPa. Crack is between $x_1 = 0$ cm and −1 cm.

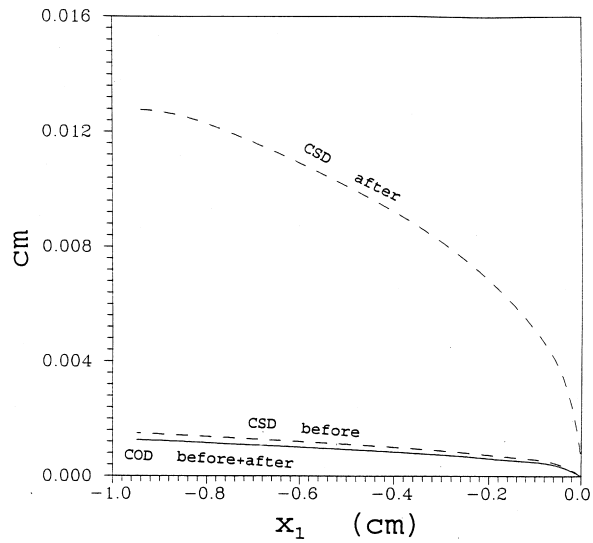

Fig. 5. COD and CSD of edge crack problem in aluminum with worn asperities, 10 mm crack length, 200 MPa applied shear stress, 40° asperity angle, $\nu = 0.3$, G = 27000 MPa. Crack is between $x_1 = 0$ and −1 cm.

yield stress τ_y simultaneously due to the uniformity of the applied load. It is obvious that the CSD increases after the wear and the COD remains the same.

Figs. 6 and 7 show the mode II and I stress intensity factors vs the applied shear stress intensity factor K_{IIapp} for steel with different coefficient of friction. As K_{IIapp} increases both K_{II} and K_I increase linearly. After passing the yielding point K_{II} jumps to a higher value and keeps increasing linearly,

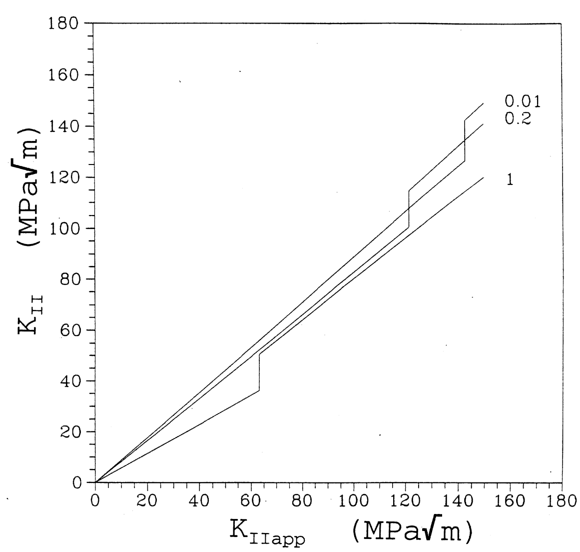

Fig. 6. Mode II shielding in edge crack problem in steel with worn asperities, 10 mm crack length, different values of coefficient of friction, 20° angle, $\mu = 1.0$, $\nu = 0.3$, G = 80000 MPa, $\sigma_y = 300$ MPa.

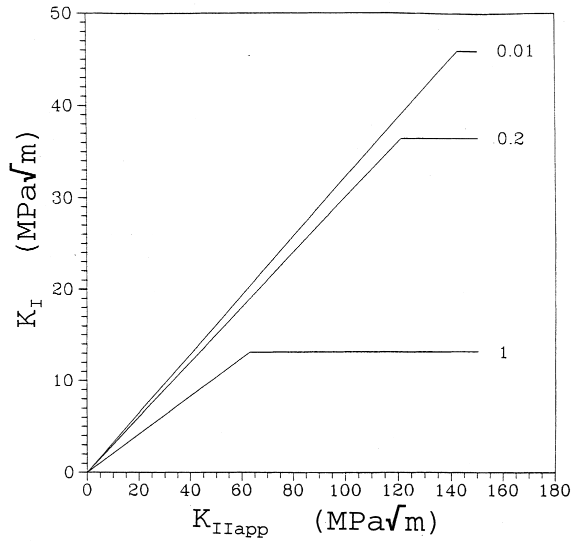

Fig. 7. Mode I enhancement in edge crack problem in steel with worn asperities, 10 mm crack length, different values of coefficient of friction, 20° angle, $\mu = 1.0$, $\nu = 0.3$, G = 80000 MPa, $\sigma_y = 300$ MPa.

however, K_I remains the same because there are no more changes in the COD after the asperities are worn flat.

A parabolic distribution of shear stress applied along the direction to crack faces with maximum at the crack mouth and zero at the tip give the results shown in Figs. 8 and 9 for steel with G = 80000 MPa, $\alpha = 20°$, $\mu = 1$ and $\sigma_y = 300$ MPa. The wear moves towards the tip as τ_{max} increases from zero in increments as shown in Fig. 8. Once again K_{IIapp} is the BEM calculated value of K_{II} for the parabolic

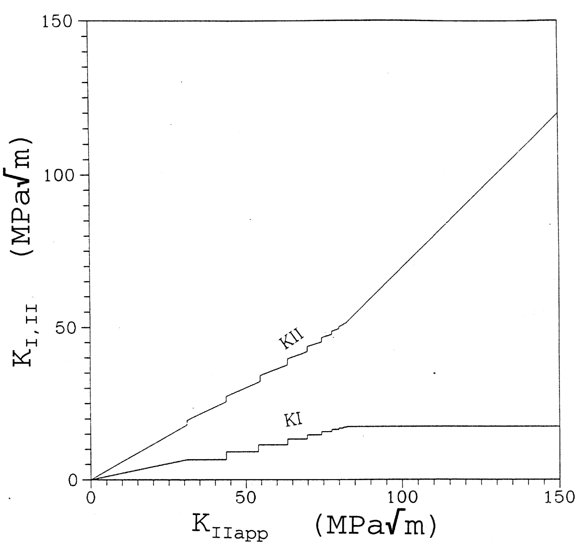

Fig. 8. Mode II shielding and mode I enhancement in edge crack problem in steel with worn asperities, 10 mm crack length, parabolic distribution applied shear stress, $\mu = 1.0$, $\nu = 0.3$, G = 80000 MPa, $\sigma_y = 300$ MPa.

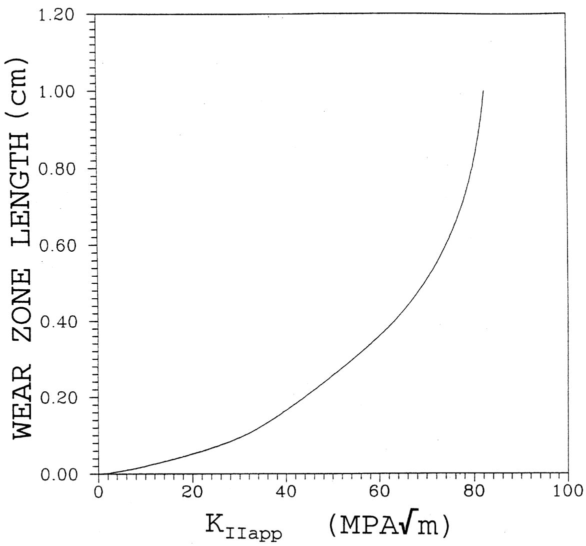

Fig. 9. Relationship between length of wear zone and maximum applied shear stress in edge crack problem in steel with worn asperities, 10 mm crack length, parabolic distribution applied shear stress, $\mu=1.0$, $\nu=0.3$, G = 80000 MPa, $\sigma_y=300$ MPa.

shear loading on the non-interfering crack. The results in Fig. 9 show a non-linear relationship between both K_I vs K_{IIapp} and K_{II} vs K_{IIapp}, which is clearly related to the non-linear dependence of wear length on K_{IIapp}.

3. Conclusions

A simple yield criterion for occurrence of wear and subsequent sliding on the worn asperity surface has been presented. The asperities are assumed to begin to wear out when the macroscopic tangential resistance stress τ_c reaches the shear yield stress τ_y where $\tau_c=\tau_y=\mu\sigma_c$ and $\tau_y = \sigma_y/\sqrt{3}$. The results show that the crack wears totally when all asperities reach τ_y simultaneously due to the uniformity of the applied load and the wear moves towards the tip as τ_{max} increases from zero due to the parabolic distribution of the applied load. Therefore, from the fracture point of view, K_{II} jumps to a higher value and keeps increasing linearly after passing the yielding point of the asperities, but K_I remains the same, i.e., the CSD increases after the wear and the COD remains the same in both cases. The results also show a non-linear relationship between both K_I vs K_{IIapp} and K_{II} vs K_{IIapp}, which is clearly related to the non-linear dependence of wear length on K_{IIapp}.

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References

Becker, W., Gross, D., 1988. About the Dugdale crack under mixed mode loading. Int. J. Fracture 37, 163–170.

Dugdale, D.S., 1960. Yielding of steel sheets containing slits. J. of Mech. Phys. of Solids 8, 100–108.

Tong, J., Yates, J.R., Brown, M.W., 1995a. A model for sliding mode crack closure, part I: theory for pure mode II loading. Engr. Fracture Mech. 52, 599–611.

Tong, J., Yates, J.R., Brown, M.W., 1995b. A model for sliding mode crack closure, part II: mixed mode I and II loading and application. Engr. Fracture Mech. 52, 613–623.

Young, L.J., 1994. A boundary element analysis of fracture surface interface for mixed mode loading problems with elastic or plastic crack tips. Ph.D. Dissertation, the Ohio State University, Columbus, Ohio, pp. 11–38.

Young, L.J., 1998. A boundary element application for mixed mode loading idealized sawtooth fracture surface. Int. J. Solids and Structures, accepted.